THE USE OF POISSON TO CALCULATE THE EFFECT OF MAGNETIZATION IN SUPERCONDUCTING MAGNETS*

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INTRODUCTION

Magnetostatic problems solved by POISSON employ current and air regions as well as regions of nonlinear permeable iron. In many problems it is customary to set the permeability of the current regions identical to that of air and to introduce a permeability table (e.g. B-H) for the iron regions. If the conductor is made of a superconducting material, setting the permeability of the current regions equal to that of air is only an approximation. The existence of surface and bulk super-currents, which act partially to shield the superconductor's interior from the penetrating field, results in the superconductor acquiring a magnetization that in some cases cannot be ignored. Magnetization in superconducting dipole magnets influences the field uniformity. This effect is quite small at high fields (H >> Hp; Hp = field at penetration) but introduces large harmonic coefficients at low fields where the magnitude of the magnetization is of the order of the applied field.

Magnetization of a superconducting material can be introduced into POISSON through a field dependent permeability table (in much the same way that iron characteristics are introduced). This can be done by representing the increasing and decreasing field characteristics by two independent magnetization curves. We have verified that superposition of a current and a magnetization table in the same region does not violate the code.

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A similar method for calculating magnetization effects was recently proposed by M. Kuchnir and E. Fisk at Fermilab. However, the method outlined here takes advantage of integrating magnetization effects into the field relaxation process and thereby avoiding some of the inaccuracies introduced by perturbation techniques. G. Morgan of BNL also has reported on the use of GFUN to calculate magnetization effects using a similar method.

We present here two examples. The first uses a linear and reversible magnetization curve for which an analytical solution is compared with that of POISSON. The second is a more realistic case where a measured magnetization curve of a superconducting cable is introduced into POISSON and results are compared with measurements.

EXAMPLE 1 - MAGNETIZATION OF A CURRENT CARRYING ANNULUS

Analytical Solution

We first analyze an arrangement (sketched below) in which a current I_0 flows, with constant current density, into an annulus of inner and outer radii a,b and returns as I_0 through the annulus center.

The 2D ring cross section area is:

$$A = \pi(b^2 - a^2)$$

The current density is:

$$J_0 = \frac{I_0}{\pi (b^2 - a^2)}$$

We make use of relations:

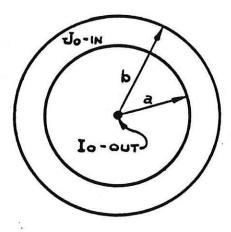
$$\oint H d\ell = \frac{4\pi}{10} I$$

H = Oersted

 $\ell = cm$

I = Amp

B = Gauss .



Then, for r < a,

$$2\pi r H_{\Theta} = \frac{4\pi}{10} I_{O}$$

$$H_{\Theta} = \frac{2I_{O}}{10r} ;$$

For
$$b > r > a$$
,

$$I = I_0 - J_0 \pi (r^2 - a^2) = I_0 \frac{b^2 - r^2}{b^2 - a^2}$$

$$2\pi r H_{\Theta} = \frac{4\pi}{10} I_0 \frac{b^2 - r^2}{b^2 - a^2}$$

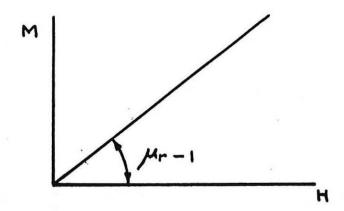
$$H_{\Theta} = \frac{2I_0}{10} \frac{b^2 - r^2}{r(b^2 - a^2)}$$

$$B_{\Theta} = \frac{2I_0}{10} \mu_r \frac{b^2 - r^2}{r(b^2 - a^2)};$$

and for r > b,

$$H_{\Theta} = 0$$
.

If $\mu_{\Gamma} \neq 1$ in the ring, then B \neq H - corresponding to a magnetization curve sketched below:



We now derive the vector potential A, using $B_{\theta} = -\partial A/\partial r$, so that it can be compared directly with POISSON's output.

Since A = constant, we choose A = 0.

$$A = \frac{2I_0}{10} \mu_r \int_{r}^{b} \frac{b^2 - r^2}{r(b^2 - a^2)} dr$$

$$A = \frac{2I_0}{10} \mu_r \left[\frac{b^2}{b^2 - a^2} \ln \frac{b}{r} - \frac{1}{2} \frac{b^2 - r^2}{b^2 - a^2} \right]$$

and on r = a

$$A = \frac{2I_0}{10} \mu_r \left[\frac{b^2}{b^2 - a^2} \ln \frac{b}{a} - \frac{1}{2} \right].$$

For r < a:

$$A = A_{r=a} - \frac{2I_0}{10} \ln \frac{r}{a}$$

$$A = \frac{2I_0}{10} \left[\mu_r \left(\frac{b^2}{b^2 - a^2} \, \ln \, \frac{b}{a} - \frac{1}{2} \right) - \ln \, \frac{r}{a} \right].$$

If we select a = 1 cm, b = 2 cm, and $I_0 = 4000$ Amp, we calculate:

In Table I below we compare numerical results for $\mu_r = 0.5$ and 1.5.

μ _r	r (cm)	A - analytical	A - Poisson	۵%
0.5	0.5	724.20	723.3	0.12
	1.0	169.68	169.28	0.24
	1.5	36.76	36.6	0.45
1.5	0.5	1063.55	1062.3	0.12
	1.0	509.04	508.15	0.17
	1.5	110.29	110.0	0.26

EXAMPLE 2 - MAGNETIZATION OF A SUPERCONDUCTING DIPOLE MAGNET

Method and Application

We construct two groups of input tables for POISSON, intended to describe the magnetization of superconductor cables used in a dipole magnet [D-12C-2]. One group of tables includes all magnetization curves, of various cable types, during a field increase and the other provides similar curves for a decreasing field.

We require magnetization curves for the identical cables used in this magnet in order to take care of variations in strand size, copper to superconductor ratio, transport current, and critical current. The magnetization curve of an entire block and not of a single turn (or cable) will be required to take care of insulation, cable compactness,

small wedges, and other non-magnetic materials, since current regions in POISSON are usually represented by a single block rather than by a collection of individual turns.

In many cables measured magnetization curves may not be available and then the use of scaling may be required. The magnetization curve should be available over a range of field extending to values as high as the short-sample limit.

Magnet D-12C-2

We next compare the measured sextupole and decapole moments of LBL-SSC model magnet D-12C-2 with those calculated by POISSON.

The inner and outer layers of the 4-cm bore two-layer magnet (Fig. 1) are made of a 23-strand and a 30-strand cable respectively, with 1.3 and 1.8 Cu/sc ratios. Stainless-steel collars over the outer layer displaced the iron to a radius of 5.57 cm. We have ignored possible saturation of the iron and therefore set the iron permeability to $\mu = \infty$ in these studies of magnetization effects. It is planned that the effect of images in iron of variable permeabilities will be checked in later work. Each individual layer has been subdivided in the computations into two parts of equal radial thickness in order to take care of the radial dependency of the current density and magnetization.

At the time this work was carried out only magnetization measurements for the inner layer were available to us. Such data took into account the existence of copper and superconductor only. We therefore took the steps necessary to scale this single magnetization curve so as to reflect the physical conditions in each of the sublayers as they exist during magnet operation. The full details of the calculations have been placed in Appendix A.

Results

A series of POISSON runs was made (total of 32) to produce data in the range of 0.1 T to 6.8 T. The first half of the runs employed magnetization tables corresponding

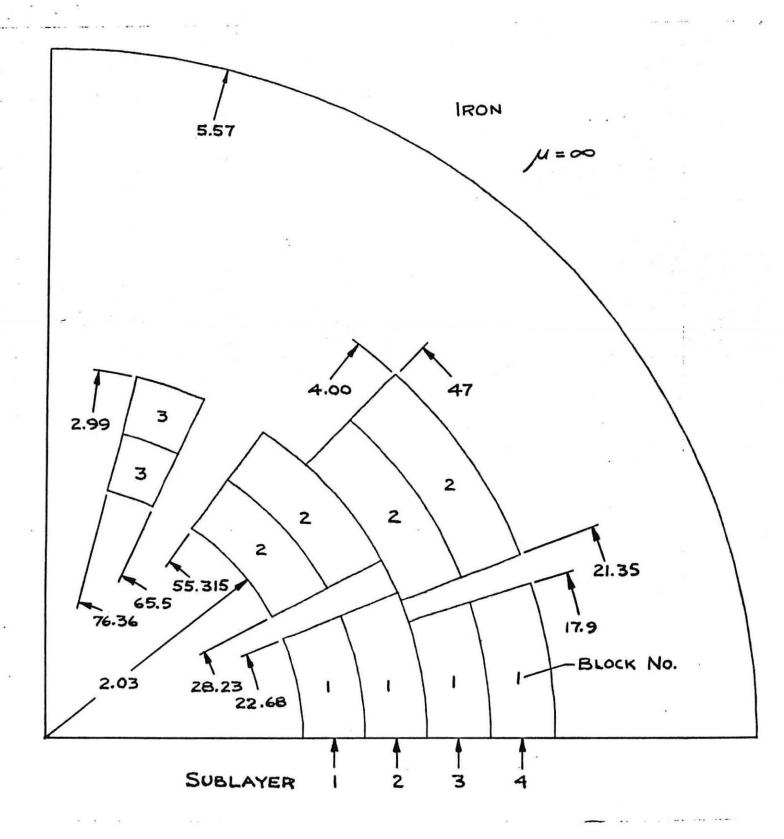


Fig. 1. Two-layer SSC type dipole used in calculations of magnetization. Numbers in current blocks correspond to the index scheme used in Appendix A.

to an increase in current, and the remainder, for the same field interval, employed magnetization tables for decreasing current. At each field level we obtained two solutions such that upon subtracting their vector potential values we were left with a vector potential that corresponds to the total field change due to magnetization effects. The differential field harmonics (up minus down) were calculated and are plotted in Figs. 2-4. The individual sextupole and decapole coefficients for a full cycle (b_2 and b_4) are plotted in Figs. 5 and 6 (all harmonic calculations were performed at 1 cm radius). Flux lines during a current increase at 0.28 T are plotted in Fig. 7. The effect of magnetization for these two curves can scarcely be distinguished. In order to suppress the dominating transport current effect we accordingly have calculated the field assuming no magnetization (μ = 1 in the current regions), and then subtracted the resulting vector potential from the vector potential at the same field when magnetization is present. Flux lines due to magnetization only are plotted in Fig. 8.

Δ Dipole for design D (4 cm)

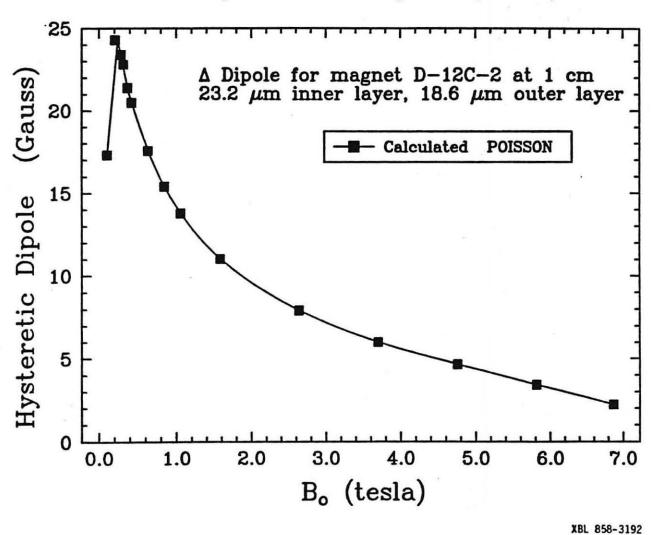


Fig. 2. Differential dipole component (up minus down) due to magnetization.

Δ Sextupole for design D (4 cm)

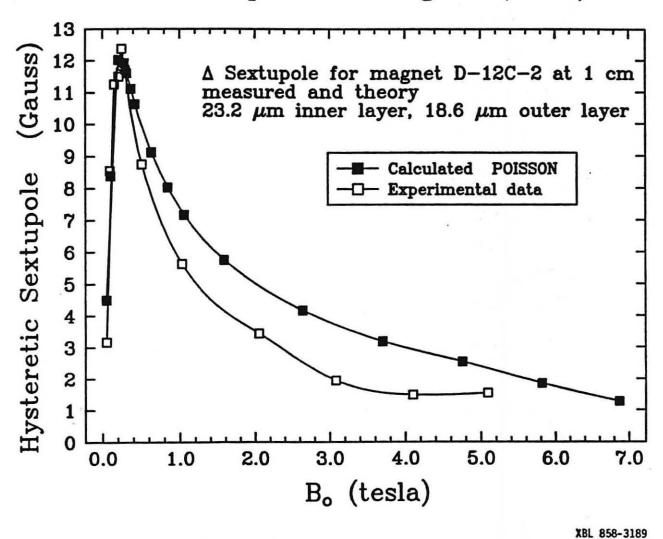
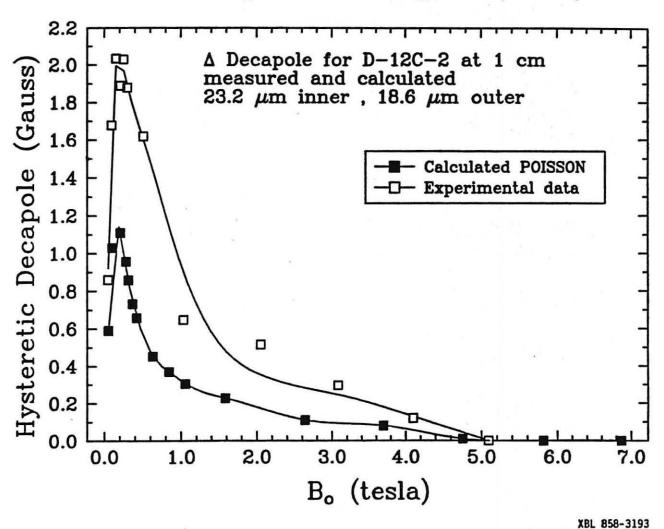


Fig. 3. Differential sextupole component due to magnetization.

Δ Decapole for design D (4 cm)



Differential decapole component due to magnetization.

D-12C-2 Sextupole

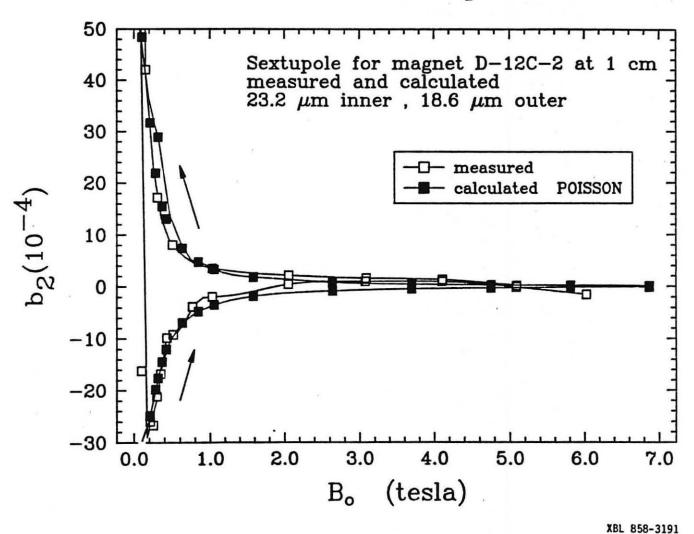
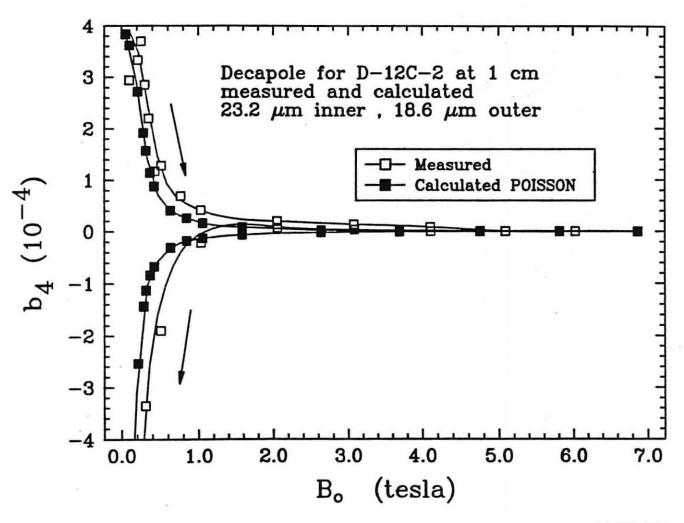


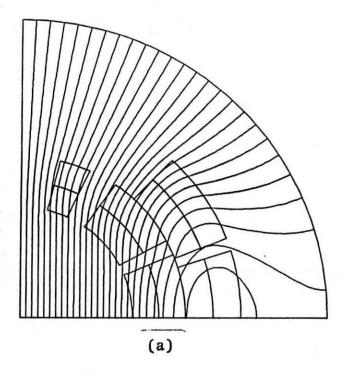
Fig. 5. Sextupole coefficient due to magnetization during a full field cycle.

D-12C-2 Decapole



XBL 858-3190

Fig. 6. Decapole coefficient due to magnetization during a full field cycle.



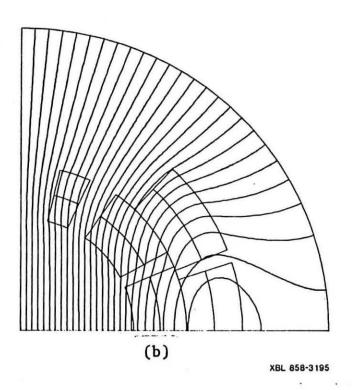


Fig. 7. Flux lines during a current increase (a) and a current decrease (b), at 0.28 T. The effect of magnetization can scarcely be distinguished.

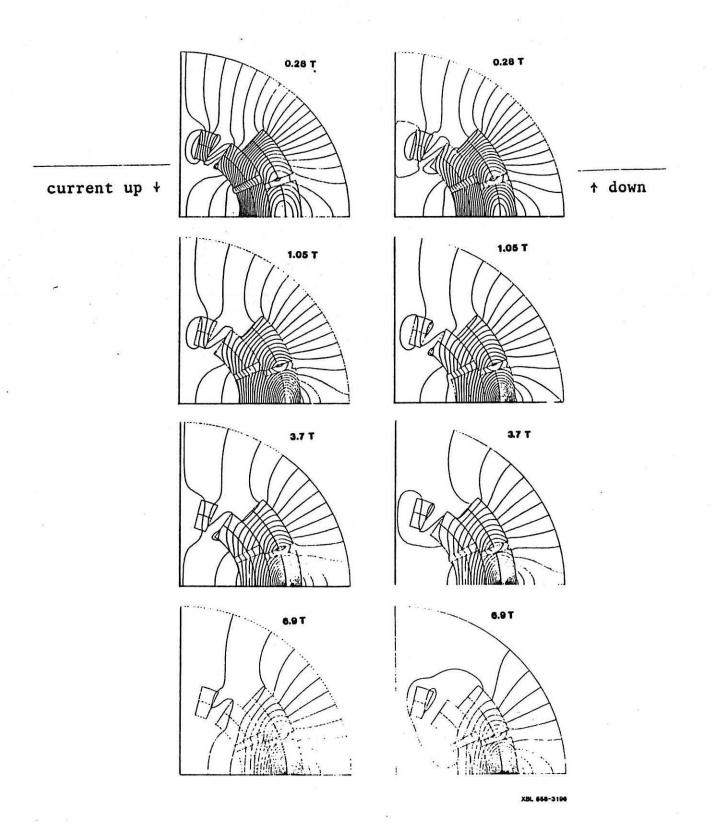


Fig. 8. Flux lines due to magnetization only, at various field levels, produced by subtracting the vector potential which includes magnetization from the one that has zero magnetization.

APPENDIX A

Scaling Procedure

We shall be using the notation F_{ij} , where i corresponds to the type of scaling and j denotes the sublayer number.

- Calculate F_{1j}, the ratio between the true copper and superconductor cross section area to that of the winding block. The amount of copper is as specified by the Cu/sc ratio for each sublayer.
- 2) Generate the critical current curve as a function of field and superpose the load line corresponding to the field at the windings. From the ratio δ between transport current and critical current (at a given field) calculate $F_{2j} = 1 \delta$. It has been assumed that this curve is the same for all sublayers. Note that below 1.0 T, F_{2j} is substantially constant (α 1.0) and is independent of field, therefore B or H can be used as an independent variable; the distinction between B and H accordingly is not critical for describing the functional dependence of F_{2j} .
- Use F_{3j} to scale for variations in filament size. This is the ratio between filament diameters.

Once scaling factors for each of the sublayers have been generated they should be consolidated into a single factor according to:

$$F_{j} = \prod_{i=1}^{3} F_{ij}$$

The single original M-H curve can now be scaled into four different curves, MF_j vs. H, each corresponding to an individual sublayer.

Inner-Layer - SC #280

$$d_f = 23.2 \; (\mu m) \; (SC \; only)$$

Strand current density at 5 T is:

$$J_c \simeq 2200 \text{ A/mm}^2$$

The strand diameter is:

$$D_s = d_f \sqrt{N_f(1+R)}$$

$$D_s = 8.077 \times 10^{-4} \text{ (m)} = 31.8 \text{ (mil)}$$

Strand cross section area:

$$A_s = \frac{\pi D_s^2}{4} = 5.124 \times 10^{-3} \text{ (cm}^2\text{)}$$

Cable cross section area (including Cu):

$$A_c = N_s \cdot A_s = 0.117847 \text{ (cm}^2)$$

Outer-Layer - SC #293

Filament diameter:

$$d_{f} = 18.6 \; (\mu m)$$

Number of filaments:

$$N_f = 434$$

$$N_{s} = 30$$

Strand current density at 5 T is:

$$J_c = 2400 \text{ A/mm}^2$$

The strand diameter is:

$$0_s = 6.484 \times 10^{-4} \text{ (m)} = 25.53 \text{ (mil)}$$

and cross section area:

$$A_s = 3.302 \times 10^{-3} \text{ (cm}^2\text{)}$$

Cable cross section area (including Cu):

$$A_c = 0.09906 \text{ (cm}^2\text{)}$$

F11

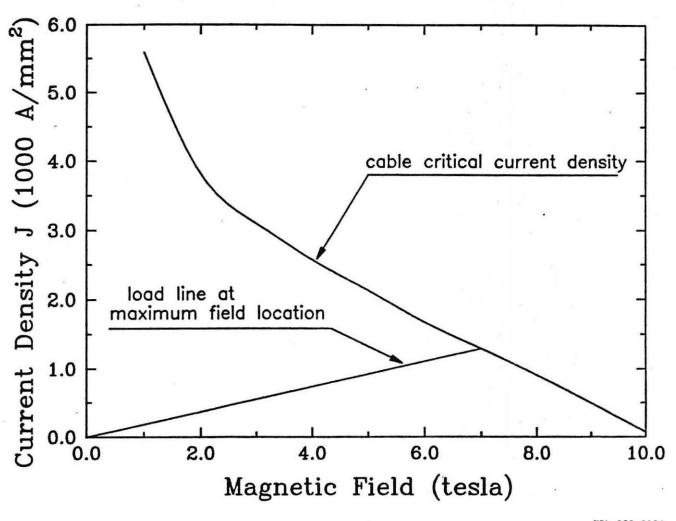
Below is a table used to calculate the scale factor F_{lj} associated with the cross sectional area ratio between the copper + superconductor and the block conductor area (see Fig. 1 in the main text).

F₂₁

We have calculated the magnetization scale factor associated with transport current. We have scaled both layers based on the J_{C} -B curve for the inner layer only. Plotted in Fig. 9 is the load line curve for magnet D-12C-2 and the cable critical current density curve (data for this curve were provided by short sample and magnetization measurements).

Sublayer	Location	Block area A _B (cm ²)	N's # of strands	Area of Cu+sc (cm ²) A's = N's As	A'S/AB	F ₁₅
1	1	0.4313	11.5	0.3535	0.820	0.825
1	2	0.5151	11.5	0.41246	0.8007	0.825
1	3	0.2064	11.5	0.17677	0.85644	0.825
2	1	0.5225	11.5	0.3535	0.6766	0.680
2	2	0.6240	11.5	0.41246	0.6610	0.680
2	3	0.2502	11.5	0.17677	0.7065	0.680
3	1	0.5115	15	0.39624	0.7746	0.790
3	2	0.7330	15	0.59436	0.811	0.790
4	1	0.5913	15	0.39624	0.67011	0.680
4	2	0.8471	15	0.59436	0.7016	0.680

Magnet D-12C-2



XBL 858-3194

Fig. 9. Critical current density curve for inner-layer cable.

B(T)	I(A)	It(A)	$\delta = \frac{I_t}{I_{ss}}$. F _{2j} = 1-8
0.5	42000	473	0.0112	0.988
1.0	32000	945	0.02937	0.9706
2.0	21110	1891	0.08957	0.9104
3.0	17216	2837	0.1648	0.835
4.0	13988	3782	0.27	0.7296
5.0	11273	4728	0.4194	0.5806
6.0	8557	5673	0.6629	0.337
7.0	6605	6605	1.0	0

We have plotted $F_{2,j}$ as a function of B in Fig. 10 and used the following curve fit.

$$F_{2j} = 1 - \delta = -5.806 \times 10^{-4} \text{ B}^4 + 5.06 \times 10^{-3} \text{ B}^3 - 2.564 \times 10^{-2} \text{ B}^2$$

-1.3295×10⁻² B + 1.0003.

Errors in F_{2j} are of the order of 4%.

We have scaled $\mathbf{J}_{\mathbf{C}}$ at low B and M at high B using the relation:

$$2\mu_0 M = 2\mu_0 \frac{2}{3\pi} \lambda J_c(H) d$$

d ≡ filament diameter

 $\lambda \equiv \text{volume fraction of SC}; 1/(1+R)$

$$2_{\mu_{_{\scriptsize{0}}}}~\textrm{M}~=~\textrm{M}^{\textrm{up}}~-~\textrm{M}^{\textrm{down}}$$

Transport current correction

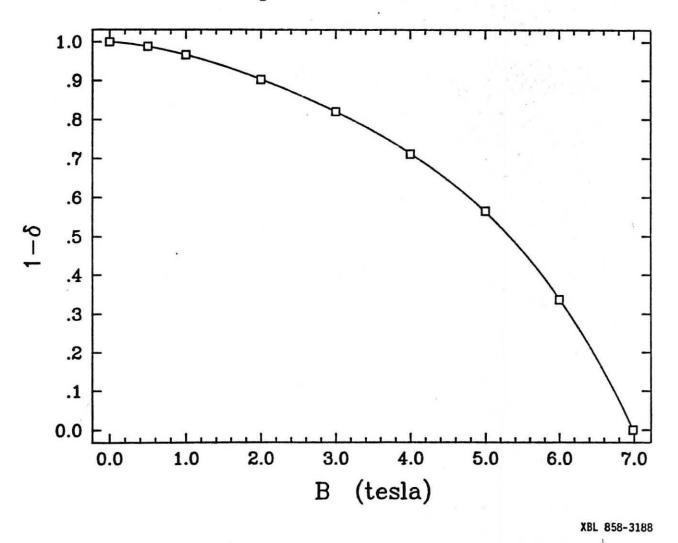


Fig. 10. Magnetization scaling factor due to transport current.

B(T)	j _c (A/mm²)	Calculated 2 _{µO} M (mT)	2 measured (mT)	measured calculated	Extrapolated (mT)
1.0	5590±12%	30.85±3.8	27.0	0.875	27.0
2.0	3810±8%	21.02±1.7	18.8	0.894	18.8
3.0	3095±8%	17.08±1.5	15.0	0.88	15.0
4.0	2565±6%	14.15±0.92			12.45
5.0	2125±3%	11.72±0.41			10.31
6.0	1670	9.21			8.10
7.0	1290	7.12			6.26

F₃₁

This scaling factor reflects variations in filament diameter. Since the measured magnetization curve of the inner layer was used, we have $F_{3}_{inner} = 1.0$. In scaling the outer layer (18.6 μ m filament diameter) from a 23.2 μ m filament diameter of the inner layer we have $F_{3}_{outer} = 18.6/23.6 = 0.802$.

Input to POISSON

A suitable input to POISSON requires a B-H or B- γ table ($\gamma = 1/\mu$). We have decided on a B- γ table. For the original M-H curve we can write:

B = H + M

Or

$$B = \left(1 + \frac{M}{H}\right) \quad H \quad .$$

With

$$\mu_{or} = 1 + \frac{M}{H}$$
; (or = original)

ORIGINAL MAGNETIZATION TABLE FROM GHOSH 23.2 μm 1.3:1 SC

<u>UP</u>				DO	<u>wn</u>	
H (Gauss)	$\gamma = \frac{1}{\mu}$					
21.8	.047		-	17.45	.03775	- ;
43.6	0.104			283.7	0.408	
148.0	0.3229			541.24	0.5921	
292.0	0.5835		4 20 7	811.87	0.70645	
476.0	0.823	3		1235.2	0.8015	
580.0	0.9868	(*C		1772.15	0.86623	
702.7	1.1218			2636.4	0.9167	
890.0	1.1793			3208.2	0.934977	
1139.0	1.2050			4430.0	0.957	
1414.0	1.2048			5914.45	0.9715	
1580.0	1,2018			7215.2	0.9787	
1846.0	1.17519			8651.2	0.98389	
2029.0	1.1621			9952.0	0.98686	
2243.0	1.1421			11622.0	0.98957	
2544.7	1.1183			13696.7	0.99169	
2815.0	1.10096			16105.5	0.99363	
3199.0	1.084			18883.5	0.99503	
3766.9	1.064			22083.5	0.99601	
4395.4	1.0495			26795.6	0.997137	
5220.0	1.03829			29661.5	0.997477	
6010.0	1.03088			32879.1	0.997918	
7010.0	1.02488			35006.6	0.998281	
8262.7	1.0197			40000.0	0.99844	
9166.0	1.016766			50000.0	0.99897	
9969.4	1.0146			60000.0	0.999325	
12219.7	1.0105			70000.0	0.99955	
14681.0	1.00796					
17775.8	1.005766					
21485.7	1.00446					
25846.0	1.003159		3 ²			
30153.8	1.00249					
35358.2	1.00205					
40000.0	1.0015					
50000.0	1.0010					
60000.0	1.00067					
70000.0	1.00045					

$$B = \mu_{or} H . \tag{1}$$

The permeability of the scaled magnetization is written as:

 $\mu_{sj} = 1 + \frac{MF_j}{H};$

10

 $\mu_{sj} = 1 + (\mu_{or} - 1) F_{j}$.

In terms of y we have:

 $\gamma_{sj} = \frac{1}{1 + \left(\frac{1}{\gamma_{or}} - 1\right) F_{j}}$

and

 $B = \left(\frac{1}{\Upsilon_{S,j}}\right) H$

Note that the parameters $\gamma_{S,j}^{}$ are H dependent.

The $\,F_{j}\,$ calculated for the four sublayers are:

 $F_1 = 0.825$ $F_3 = 0.633$

 $F_2 = 0.680$ $F_4 = 0.545$

at low field strengths, but at higher fields the factors F_{2j} that contribute to these quantities will introduce some field dependence. The original and scaled magnetization curves are included here both in tabular form, and as plotted curves, Fig. 11.

Several restrictions in input entries to POISSON are noted.

	Statement is a statement of the little	4	n e da nde		The second of the second
(4)		<u>U</u>	<u>P</u>		
В	$\gamma = \frac{1}{11}$	В	$\gamma = \frac{1}{\mu}$	В	$\gamma = \frac{1}{\mu}$
0.0	$^{\mu}$	0.0	1.130	0.0	1.10
- 0 v- 1 12 15	TO SERVICE OF STREET	30 20	1.130643		1.102060
979.327	1.163043 1.162809	1007.39	1.130461	1033.52 1283.21	1.102080
1216.02	1.162809	1250.82	1.128627	1435.68	1.101921
1361.53		1399.93	1.112504	1696.38	1.088198
1619.51 1796.15	1.139848 1.129636	1659.32 1837.08	1.104471	1875.18	1.082029
2013.52	1.113972	2053.85	1.092096	2091.40	1.072487
2323.50	1.095199	2362.38	1.077176	2398.58	1.060921
2603.03	1.081430	2640.29	1:066171	2674.97	1.052347
2995.66	1.067878	3031.40	1.055289	3064.67	1.043831
3581.36	1.051808	3613.97	1.042317	3644.33	1.033633
4225.98	1.040090	4255.76	1.032813	4283.48	1.026129
5063.13	1.030982	5090.70	1.025399	5116.37	1.020254
5863.74	1.024944	5889.45	1.020470	5913.38	1.016340
6872.32	1.020035	6896.51	1.016455	6919.04	1.013146
8134.28	1.015788	8156.85	1.012977	8177.86	1.010374
9044.95	1.013383	9066.22	1.011005	9086.03	1.008801
9854.98	1.011610	9875.09	1.009550	9893.81	1.007640
12119.7	1.008250	12137.3	1.006790	12153.7	1.005435
14591.1	1.006159	14606.9	1.005071	14621.6	1.004060
17698.5	1.004365	17712.1	1.003596	17724.8	1.002880
21415.5	1.003278	21427.8	1.002700	21439.3	1.002163
25788.5	1.002231	25798.6	1.001838	25808.0	1.001473
30103.2	1.001680	30112.1	1.001384	30120.4	1.001109
35312.5	1.001295	35320.5	1.001067	35328.0	1.000855
39963.3	1.000917	39969.8	1.000756	39975.8	1.000606
49976.1	1.000479	49980.3	1.000395	49984.2	1.000316
59988.7	1.000188	59990.7	1.000155	59992.6	1.000124
70000.0	1.000000	70000.0	1.000000	70000.0	1.000000
		DO	WN	·	(Fee
33		1 20			1
	_ 1	D D	1	D	
В	$\gamma = \frac{1}{\mu}$	В	$\gamma = \frac{1}{\mu}$	В	$\Upsilon = \frac{\pi}{\mu}$
0.0	$\gamma = \frac{1}{\mu}$.75	B 0.0	$\gamma = \frac{1}{\mu}$.78	B 0.0	$\frac{1}{0.80}$
0.0	.75	0.0	.78 ^µ	0.0	0.80
0.0 1486.99	.75 .8306738	0.0 1442.73	.78 .78	995.442	0.80 L. 8155876
0.0 1486.99 1997.15	.75 .8306738 .8873403	0.0 1442.73 1957.60	.78 .8561532 .9052652	995.442 1401.53	0.80 .8155876 .8813218
0.0 1486.99 1997.15 2832.95	.75 .8306738 .8873403 .9306197	0.0 1442.73 1957.60 2798.41	.78 .8561532 .9052652 .9421079	995.442 1401.53 1920.79	0.80 .8155876 .8813218 .9226174
0.0 1486.99 1997.15 2832.95 3390.97	.75 .8306738 .8873403 .9306197 .9461023	0.0 1442.73 1957.60 2798.41 3358.84	.78 .8561532 .9052652 .9421079 .9551504	995.442 1401.53 1920.79 2766.24	0.80 .8155876 .8813218 .9226174 .9530616
0.0 1486.99 1997.15 2832.95 3390.97 4592.43	.75 .8306738 .8873403 .9306197 .9461023 .9646310	0.0 1442.73 1957.60 2798.41 3358.84 4563.88	.78 .8561532 .9052652 .9421079 .9551504 .9706649	995.442 1401.53 1920.79 2766.24 3328.94	.8155876 .8813218 .9226174 .9530616 .9637315
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531	995.442 1401.53 1920.79 2766.24 3328.94 4537.30	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9892948 .9913348	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9892948 .9913348	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923	0.0 995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9957062 .9967649
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9892948 .9913348 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649 .9975294 .9980694 .9986757
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4 32923.9	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968 .9979968	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4 32916.0	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9913348 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1 29695.1	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9957062 .9967649 .9975294 .9980694 .9986757
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4 32923.9 35044.8	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968 .9979968 .9982883 .9986408	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4 32916.0 35038.1	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887 .9988793	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1 29695.1 32908.7	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649 .9975294 .9986757 .99868686 .9991015
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4 32923.9	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968 .9979968	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4 32916.0	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9913348 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1 29695.1 32908.7 35031.9	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649 .9975294 .9980694 .9980694 .9986757 .9988686 .9991015
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0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4 32923.9 35044.8 40036.7 50023.9	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968 .9982883 .9986408 .9989090 .9990839 .9995224	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4 32916.0 35038.1 40030.2 50019.7	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887 .9985887 .9988793 .9991006 .9992448	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1 29695.1 32908.7 35031.9 40024.2 50015.8	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9967649 .9975294 .9980694 .9986757 .9988686 .9991015 .9992790
0.0 1486.99 1997.15 2832.95 3390.97 4592.43 6055.27 7341.95 8764.78 10057.5 11718.7 13786.1 16184.6 18954.3 22148.2 26849.4 29712.4 32923.9 35044.8 40036.7 50023.9 60011.3	.75 .8306738 .8873403 .9306197 .9461023 .9646310 .9767445 .9827359 .9870416 .9895065 .9917520 .9935144 .9951109 .9962648 .9970806 .9979968 .9982883 .9986408 .9989090 .9990839 .9995224 .9998121	0.0 1442.73 1957.60 2798.41 3358.84 4563.88 6030.52 7319.67 8744.81 10039.0 11701.7 13770.4 16170.7 18941.9 22136.8 26839.9 29703.4 32916.0 35038.1 40030.2 50019.7 60009.3	.78 .8561532 .9052652 .9421079 .9551504 .9706649 .9807531 .9857268 .9857268 .9892948 .9913348 .9931918 .9946482 .9959668 .9969194 .9975923 .9983484 .9985887 .9988793 .9988793 .9991006 .9992448 .9996063 .9998450	995.442 1401.53 1920.79 2766.24 3328.94 4537.30 6007.48 7298.93 8726.23 10021.7 11685.8 13755.8 16157.8 18930.3 22126.2 26831.1 29695.1 32908.7 35031.9 40024.2	.8155876 .8813218 .9226174 .9530616 .9637315 .9763511 .9845151 .9885279 .9914019 .9930431 .9945362 .9957062 .9957062 .9967649 .9975294 .9980694 .9986757 .9988686 .9991015 .9992790 .9993945 .9996845

Magnetization Curves D-12C-2

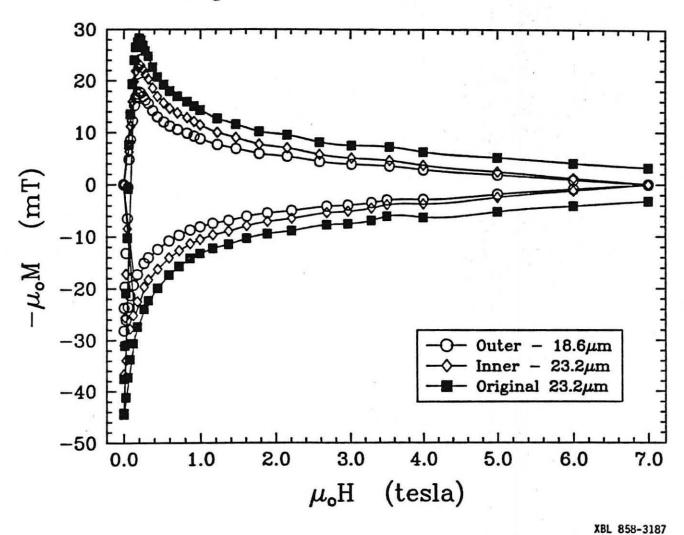


Fig. 11. Some of the magnetization curves used in the present calculations.

POISSON is restricted in the number of permeability tables it can accept. Excluding the built-in table, up to three additional tables can be entered. This condition can be relaxed through the use of the POISSON stacking factor. We decided, however, to limit magnetization entries in the present work to three tables and therefore substituted F_2 for F_3 .

The second and the most troublesome entry to POISSON arises from following this technique at low fields (H < 300 Gauss), where double values of γ vs. B can then occur. Since POISSON does not support B entries in a permeability table which are less than zero we were forced to approximate the table entries at low fields. An adjustment to the M-H curve at low field (Fig. 12) was made so that at H = 0, M = 0 (Fig. 13). This was done by drawing a linear asymptote from the origin to the upper curve and adjusting the lower curve analogously by drawing a linear curve from the origin over the same field interval.

Magnetization Curves D-12C-2

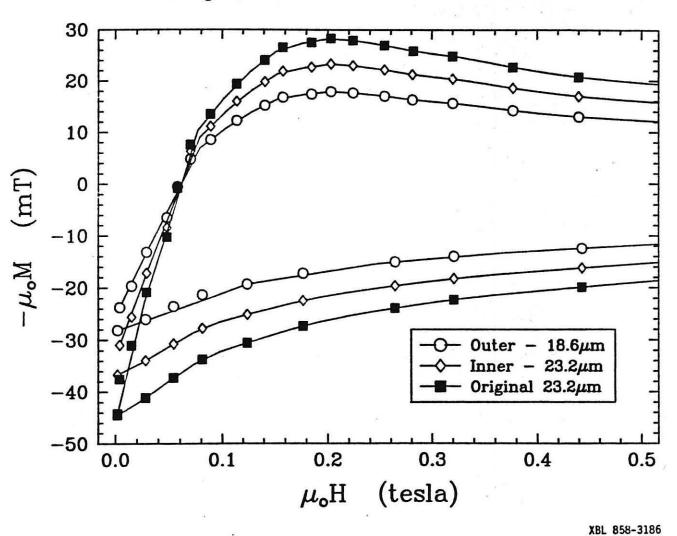


Fig. 12. The low field magnetization values as derived from scaling.

Magnetization Curves D-12C-2As used in POISSON

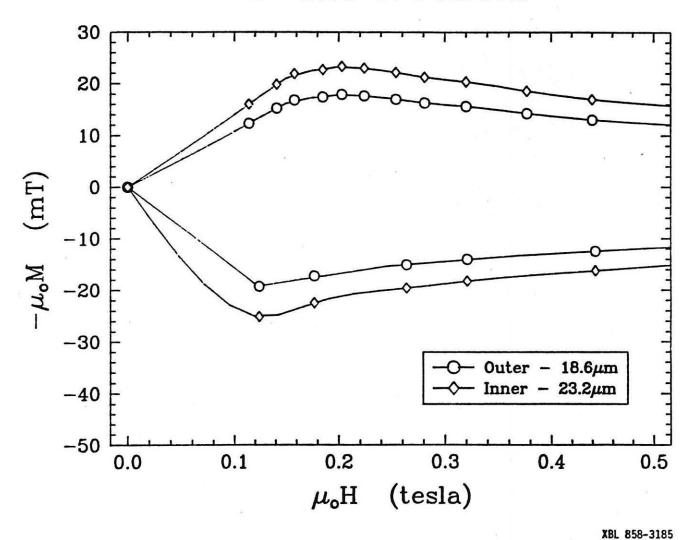


Fig. 13. Low field magnetization adjustment as introduced to POISSON.